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ABSTRACT

This publication presents teaching strategies to develop understanding of fractions as sensible numbers. Written for the mathematics teacher in grades K-5, it gives suggestions for teacher-led discussions and structured exploratory activities. The activities are conducted with manipulative materials that emphasize the important characteristics and properties of fractions. Sections focus on: the idea that fractions are rational, a developmental sequence for teaching fraction concepts, concrete models for teaching fractions, models for fractions greater than one, models for equivalent fractions, and teaching to improve problem-solving skills. Appended are pages of dot paper, centimeter graph paper, pattern blocks, and a tangram. References are also included. (MNS)

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Strategies for Teaching Fractions

Kindergarten Through Grade 5

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On The Cover

The cover design illustrates some of the infinite ways to represent fractions. The design can be used for investigating concepts, such as $\frac{1}{2}$. The four small blue rectangles cover $\frac{1}{2}$ of the large rectangle. It follows that the part of the rectangle not blue is also $\frac{1}{2}$. The small squares illustrate the fraction $\frac{24}{48}$ as well as $\frac{1}{2}$, indicating that $\frac{24}{48}$ and $\frac{1}{2}$ are equivalent fractions. Each blue rectangle is $\frac{1}{8}$ (or $\frac{1}{4}$ of $\frac{1}{2}$) of the large rectangle. Using this kind of illustration, the teacher may help the student understand the process of multiplying fractions.

STRATEGIES FOR TEACHING FRACTIONS

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Texas Education Agency
201 East 11th Street
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Foreword

This publication presents teaching strategies to develop understanding of fractions as sensible numbers. Written for the mathematics teacher K-Grade 5, it gives suggestions for teacher-led discussions and structured exploratory activities. The activities are conducted with manipulative items that emphasize the important characteristics and properties of fractions.

We hope this publication will assist school administrators and instructional personnel in improving mathematics instruction for students.

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This publication was developed by staff members in the Office of the Deputy Commissioner for Planning, Research, and Curriculum. It was written by Janie Schielack while she was an Agency education specialist. She is now with the mathematics department of Texas A&M University.

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Fractions Are Rational

Do fractions make sense? If a student can see the relationships between ideas about fractions and other familiar mathematical ideas such as whole numbers, geometry, and measurement, then fractions will make sense as numbers.

One-sixth is less than one-fifth for the same reason five is less than six: because each number represents a specific amount. Students who have developed visual images of fractions know that larger denominators mean smaller pieces. The answer to $\frac{1}{3} + \frac{1}{2}$ is greater than either $\frac{1}{3}$ or $\frac{1}{2}$ for the same reason that the length around a triangle (the perimeter) is longer than any of its sides—because adding two positive amounts makes a greater amount. Activities with concrete materials and time for discussion with the teacher allow students to make these connections and to form a concept of fractions that fits into the common sense knowledge they have of the world around them.

Often students struggle over deciding whether $\frac{1}{6}$ is greater than or less than $\frac{1}{5}$, even though common sense dictates that a pie cut into six equal pieces gives smaller servings than the same pie cut into five equal pieces. Students commonly become confused when learning addition of fractions. They frequently make the error $\frac{1}{3} + \frac{1}{2} = \frac{2}{5}$, even though experience has shown that adding some clay to a lump of clay on the table increases the total amount of clay. For students who have not formed the connection between mathematical symbols and their physical representations, tasks such as ordering and adding fractions consist of memorizing rules based on nothing more than teacher or textbook authority.

A teacher should try to illuminate as many of the connections between the mathematical symbols and their physical representations as possible. From these connections, the student can build a working knowledge of numbers, such as fractions, based on common sense. A comment by mathematics educator W. W. Sawyer explains:

> "The depressing thing about arithmetic badly taught is that it destroys a child's intellect and, to some extent, his integrity. Before they are taught arithmetic, children will not give their assent to utter nonsense; afterwards they will. Instead of looking at things and thinking about them, they will make wild guesses in the hope of pleasing a teacher or examiner."

To help you as a teacher promote independent thinking rather than mindless acceptance, this publication presents teaching strategies to develop understanding of fractions as sensible numbers.

In *An Agenda for Action*, the National Council of Teachers of Mathematics recommends that "problem solving be the focus of school mathematics in the 1980s." Recent analysis of research on the ability of elementary school children to solve problems lists characteristics of a good problem solver, including:

- the ability to understand mathematical terms and concepts
- the ability to note likenesses, differences, and analogies
- the ability to visualize and interpret quantitative or spatial facts and relationships
- the ability to generalize on the basis of few examples
- the ability to switch methods readily

The following suggestions for teaching fractions are built around the idea of strengthening these problem-solving characteristics.

Developmental Sequence For Teaching Fraction Concepts

Many children entering school are familiar with representations of halves, fourths, and thirds. As a teacher, you must take this familiarity as a starting point, expand it, develop it, and make it more precise. To expand and develop their concept of part of a whole, students need to participate in a planned sequence of teacher-led discussions and structured exploratory activities with manipulative models that emphasize the important characteristics and properties of fractions.

The following is a concise list of four basic ideas that lead to the understanding of fractions:

- identifying whole objects and complete collections of objects
- counting the parts of whole objects or members of a collection of objects and being able to tell in what ways the parts are all the same
- using a pair of numbers to describe a given part of a unit
- using fraction names and symbols to represent the number pairs that describe a given amount of a unit

Pre-Fraction Concepts: Wholes and Parts

The first step in the sequence to develop understanding of fractions is the ability to recognize a unit and its parts. Primary teachers, especially, should take every opportunity to have students identify whole objects and collections of objects. Classroom settings offer many spontaneous opportunities for students to identify units such as the whole room, the set of girls in the class, the container of blocks, or the carpeted area of the floor. You should also develop some planned lessons in which appropriate discussions of units arise. Units represented by continuous or connected wholes include:

- an orange
- a candy bar
- a cake
- a quart or liter of liquid
- an hour, a day, a week, or a year
- a circular object (pie)
- a stick, ruler, or rod
- a wall or floor
- a window

Units represented by collections include:

- a dozen eggs
- pages of a booklet
- a tray of ice cubes
- a deck of cards
- a box of envelopes
- a bag of candy with all pieces alike
- a bag of candy with different kinds of pieces
- a collection of pictures
- sets of hats, dresses, shirts, shoes
- a school of fish
- parts of a car or bicycle
- a bouquet of flowers (all alike or of several kinds)
- houses on a street
- letters of the alphabet
- fingers on a hand
- windows in a house
- a collection of dolls or coins
- members of a team
- a set of teeth

Since the meaning of a fraction depends upon the unit it refers to, students should have experience in working with as many representations of the unit as possible, including entire objects, collections, quantities, distances, and lengths. Students should begin to realize that a unit is completely arbitrary; it can be whatever a person wants it to be.

One discussion activity that emphasizes the importance of identifying the unit uses a box of soda crackers. The unit in this activity may be identified first as the entire box of crackers, with the four packages inside as parts of the whole. (Students who have learned some of the fraction vocabulary may want to use fourths to describe the parts.) What happens if we change the definition of the unit to be one of the packages inside? Then the entire box of crackers represents four units, and the crackers inside the packages are parts of units. Depending on your students' levels of understanding, continue this activity by defining the unit as one of the whole crackers inside a package and then as one of the single

cracker squares. Your questions should direct students to recognize that each time the unit changes, the numbers used to describe each of the other parts (the whole box, the single cracker) change.

As they progress in the recognition of units and parts of units, students should begin to make some conclusions about whether or not the parts of a unit are related and, if so, how they are related. With parts of an orange or candy bar, the pieces might be irregularly shaped so that they are not equal amounts. In a floor or window, the parts might be equal in size and shape.

When discussing the members of a collection of objects as parts of a whole, students should be led to consider characteristics other than shape and size. Each member of a set of red felt geometric shapes is the "same" as any other member because of its color. Here again, the unit itself determines which characteristics of the unit's parts are important.

Language, Then Symbols

Language development is important in understanding fractions. Students benefit from using the word form for fractions for a period of time before being presented the numerical form. When oral names of fractions are introduced before written symbols, there is a marked reduction in the number of reversal errors ($\frac{4}{3}$ for $\frac{3}{4}$) in later work. Learning theories from the language arts can be applied to suggest mathematics should be presented in ordinary language before superimposing mathematical notation.

Let's consider more closely the approach of using what is known to be effective in teaching reading to develop strategies for teaching mathematics. Most elementary teachers agree students learn to read words they know and use in speech more easily than those they have not heard or seen. Learning to read a word like **salamander** before having any experience with or a visual image of that animal requires the student to memorize a word symbol without an anchoring visual or sound image. Consequently, it will be easily forgotten. Learning to read and use symbols in mathematics is a similar task. The mathematical symbols are more easily learned and remembered if they represent ideas that have already been visualized and put into meaningful language. Therefore, in fractions, the development of the concepts of wholes and parts of wholes should be followed by building language patterns that describe specific parts of a whole.

Children have a distinct notion of fairness at an early age. Divide a candy bar among a group of three students by giving a tiny piece of the candy bar to one student, a very large piece of the candy bar to the second student, and the remainder of the candy bar to the third. There will be an immediate response, probably from the student with the tiny piece, that the portions are not fair.

"But doesn't everyone have **one** piece of the candy bar?" you ask. The student will claim the pieces must represent the same amount for the portions to be fair. Starting with a fresh candy bar, divide it into three equal pieces and ask the students if they would like one piece of candy bar. This time, one of the

students can be given a small corner broken off of one of the pieces. The discussion that follows should emphasize that using only one number to describe a piece of the candy bar does not provide enough information. The student should begin to look for a more precise way to describe a certain amount of the candy bar. With direction, they should begin to use a pair of numbers, "**one** of the **three** equal parts," to describe each person's share. This concept is what distinguishes fractional numbers from whole numbers. A fraction is a relationship between two quantities.

In the primary grades, activities to develop fraction concepts should emphasize practicing this language—using a pair of numbers to describe a given portion of a unit. Students in the higher elementary grades who are having difficulty with fraction symbols also need practice using this language to strengthen the important connection between the physical representations of fractions and the mathematical symbols. Here again, students should be encouraged to use this language whenever possible in natural settings with specific lessons planned as needed.

Students should learn to use this language comfortably to answer the question, "How much?" For example, they should give answers such as "two of the five equal parts," "three of the four marbles in the bag," "one of the 10 big marks on the meter stick." Then the vocabulary **two fifths, three fourths,** and **one sixth** will become a verbal shorthand for the longer phrases. Similarly, the written forms of $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{1}{6}$ become a label that stands for the verbal descriptions. A special effort is needed to teach the child the connection between the oral forms and written forms for fractions as well as between the written forms and the concrete models. If students have a strong visual image of fractions developed through the use of concrete materials and a strong verbal model developed through the use of meaningful language, the fraction symbols can be presented as a reasonable way to record the number pair necessary to describe a given amount of a unit.

This sequence for developing understanding of fractions follows the sequence for strengthening the problem-solving characteristic of understanding mathematical terms and concepts:

- nonverbal experience with things and people
- verbalization—oral language development
- recording and reading about quantitative ideas in the symbols of ordinary language
- extending ordinary language with mathematical symbols

The written symbols simply become a code for something the student already knows.

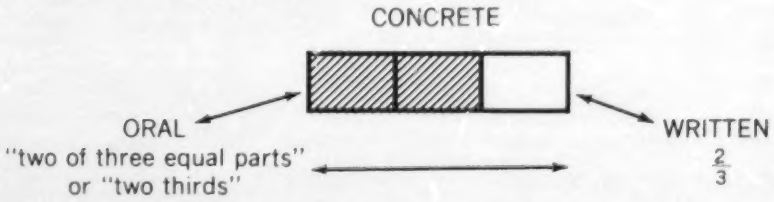
Concrete Models For Teaching Fractions

Most elementary school educators agree that manipulative materials are useful in teaching new concepts. Work with manipulatives develops the problem-solving characteristics of noting likenesses and differences and visualizing and interpreting mathematical relationships. Purely symbolic treatments of mathematical topics are not as effective as the use of activities in which manipulative materials are used.

Fractions can be introduced in kindergarten and Grade 1 if manipulatives are used and if the oral work does not involve symbols. The next step, of course, is to guide the students to recognize the connection between the manipulatives, the language, and the symbols.

Beginning fraction concepts should be based on natural settings as often as possible. As students mature in their understanding of fractions, they will need structured activities involving manipulative materials designed to advance their fraction concepts to a more abstract level.

There is no specific step-by-step order of instruction for presenting activities with concrete materials. For each fraction idea, students must progress through the levels of understanding as they first use concrete materials and language, then connect the concrete materials and language to written symbols, and finally use the written symbols.



Some students will achieve the symbolic level of understanding more quickly than others; some are able to make generalizations from fewer examples than others. Most young children will need emphasis on concrete experiences and language development. A few with strong visual images of fractions will be ready to connect the written symbols to the images. Their interest in using manipulatives can be maintained through more challenging questions and letting them explore the written symbols with the manipulatives. If you are working with older students who seem to lack the basic understanding needed for working with fractions, provide them opportunities to develop that understanding. Don't require them to memorize "senseless" rules to use in problem situations they do not understand.

Results of the second National Assessment of Educational Progress in mathematics showed lack of conceptual understanding to be a major cause of weak computational skills with fractions. The results indicated students need

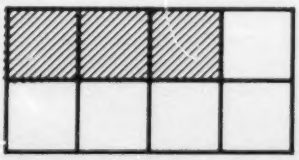
a more extensive exposure to fraction concepts before teachers introduce computing with fractions. You must decide the best path to take with older students who show a lack of understanding. Ask them to use concrete materials or drawings to explain some fraction situations they are supposed to understand. Then observe their responses. Let this guide you in the best choices of activities to use to correct or expand their understanding.

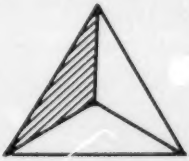
Use a wide range of concrete materials to represent fractions so that students do not make a "misconnection," linking the fraction concept to an irrelevant characteristic of one particular material. If only pie shapes are used to teach fractions, students may not understand that the fraction symbol can also be used to describe rectangular regions. Or, if only connected wholes are used, they may not see the application of fraction concepts to parts of a set. The more varied the visual representations students collect for a fraction, the broader their understanding will be of the symbol's abstract meaning.

Geometric (Area) Representations of Fractions

The following activities suggest ways of teaching about fractions. Work through the activities and experience the suggested materials yourself so you can decide which ones are most appropriate for you and your students. This process may also make you aware of other materials and activities that could be adapted to teaching fractions.

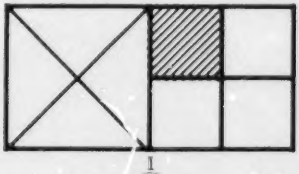
In working with geometric figures to represent fractions, the region of the figure usually is considered to be the whole. It is separated into parts by lines or colors. To introduce the use of geometric regions to represent fractions the students have experienced earlier in normal settings, first use regions that are divided into sections of the same size and shape.





For students who need a challenge, or for later instruction to extend understanding of **equal parts,** the regions should be divided into parts that are of equal size but different in shape:

$\frac{1}{4}$



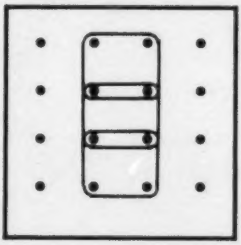
$\frac{1}{8}$

These shapes may be made of paper so that students can cut them apart and show their equivalence. (One of the triangular sections in the first figure could be cut in half and reassembled into a rectangle that is congruent to the rectangular sections.)

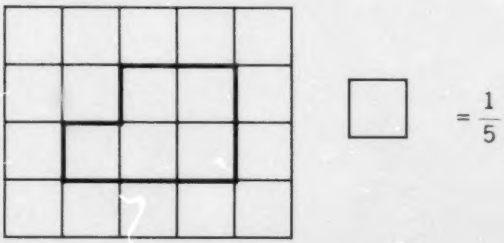
Geometric regions can be represented with felt or paper cutouts, geoboards, graph paper, pattern blocks and tangrams, and colored cubes in the following ways:

- Paper shapes: Students can fold circular or rectangular shapes into halves, fourths, eighths, or other designated fractional parts. Finding thirds, fifths, and sixths are more challenging folding tasks.

- Geoboards: A geoboard can be made from a square pegboard with golf tees placed in the holes. A geometric region can be represented on the geoboard and divided into sections with colored rubber bands. Activities with the geoboard should include asking students to name (orally or in writing) the fraction that describes the sections shown on the geoboard as well as asking students to show on the geoboard a picture for a given fraction symbol. If space and materials are unavailable for your students to use geoboards, the same activities can be done with laminated dot paper (Appendix A) and crayons by drawing loops around the dots to represent the rubber bands around the pegs on a geoboard.

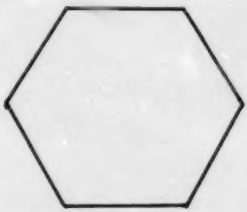


- Centimeter graph paper: Ask students to outline a figure on centimeter graph paper (Appendix B) so that one of the centimeter squares equals $\frac{1}{4}$, $\frac{1}{5}$, or any other designated unit fraction.

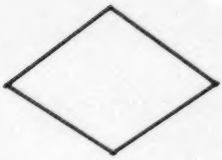


- Pattern blocks and tangrams: Pattern blocks (Appendix C) consist of a set of four different shapes—triangle, diamond, trapezoid, and hexagon. The shapes are often color-coded so they can be referred to by color if a student has not yet learned the geometric vocabulary. A tangram (Apendix D) is a Chinese puzzle consisting of seven pieces cut from a square.

In fraction activities with these materials, you or the student can identify a particular shape to be the unit. Then determine the numerical descriptions of the other shapes based on that unit. For example, using the pattern blocks, name the hexagon as the unit.



It takes two trapezoids to cover the hexagon so each trapezoid is $\frac{1}{2}$ of the hexagon.



It takes ____ diamonds, to cover the hexagon, so each diamond is ____ of the hexagon.

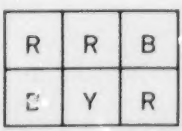
It takes____triangles to cover the hexagon. Each triangle is ____ of the hexagon.

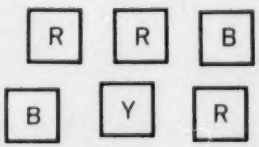
Similar activities may be done with other pattern block shapes or tangram pieces as units. Notice that if a smaller piece is chosen for the unit, students will need to use some whole numbers to decribe the larger pieces. For example, if the square in the tangram pieces is chosen to be the unit, the small triangle would represent $\frac{1}{2}$. The middle-sized triangle and the parallelogram would each represent 1, and the large triangle would be 2 units.

- Colored cubes: Three dimensional shapes can be made with cubes colored red, blue, yellow, green, etc. Direct the students to build a shape from a certain collection of cubes. Then have the students answer questions involving fraction ideas. For example, have students arrange one layer of cubes using three red cubes, two blue cubes, and one yellow cube. Ask students to use fractions to answer the following questions:

- How much of the shape is red?
- How much is blue?
- How much is yellow?
- How much of the shape is red or blue?
- If the shape is considered a unit (worth 1), then how much is each cube worth?

These activities can also be used to represent the connections between parts of wholes and parts of a set. With the cubes arranged close together, they represent a geometric solid; with the same cubes spread apart, the represent a collection of objects.



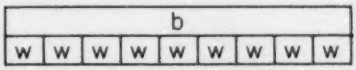


The students can see that the same fractions are used to describe amounts in either arrangement.

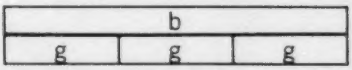
Linear (Measurement) Representation of Fractions

Fractions are often used in describing linear measurements. Whether measuring with a ruler marked in centimeters or inches, a yardstick, a meter stick, or a popsicle stick, lengths seldom come out in whole numbers of units. Fractions can be used to describe measurement amounts less than the unit being used. Materials based on measurement representations include colored rods, fraction bar kits, and number lines.

- Colored rods: Sets of colored rods consist of several wooden or plastic rods of 10 different lengths from 1 cm to 10 cm, with each length being a different color. In fraction activities with the rods, one of the lengths (or colors) is designated as the unit, and fractions are used to describe the lengths of the other rods. For example, if the 9 cm rod is chosen to be the unit, the students are asked to match the 9 cm rod with "trains" of other rods. The 9 cm rod can be matched to nine 1 cm rods or to three 3 cm rods.

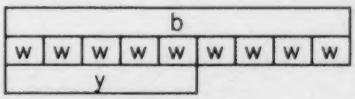


It takes nine whites to make one blue rod. So a white rod is $\frac{1}{9}$ of a blue rod.



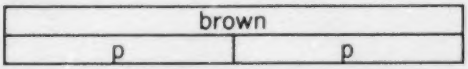
It takes ____ greens to make a blue. So a green is ____ of a blue.

Some of the rods will not have a unit fraction description (a fraction with a numerator of 1), depending upon the unit chosen. As in this example, a 9 cm rod cannot be matched to a whole number of 5 cm rods. But if the 5 cm rod is placed next to the train of 1 cm rods, then the 5 cm rod can be described in terms of the 1 cm rods.

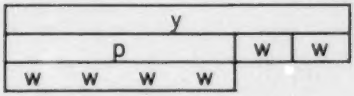


It takes five whites to make a yellow. One white is $\frac{1}{9}$ of a blue. So a yellow is $\frac{5}{9}$ of a blue.

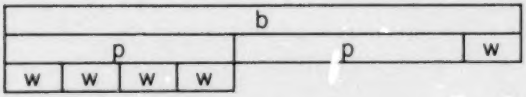
For students who are ready to investigate fractions larger than one, the activities can include describing rods that are longer than the unit rod. If the 4 cm rod is chosen as the unit, then the other rods can be described by comparing them to the unit:



If purple is 1, then brown is 2.



If purple is 1, then white is $\frac{1}{4}$



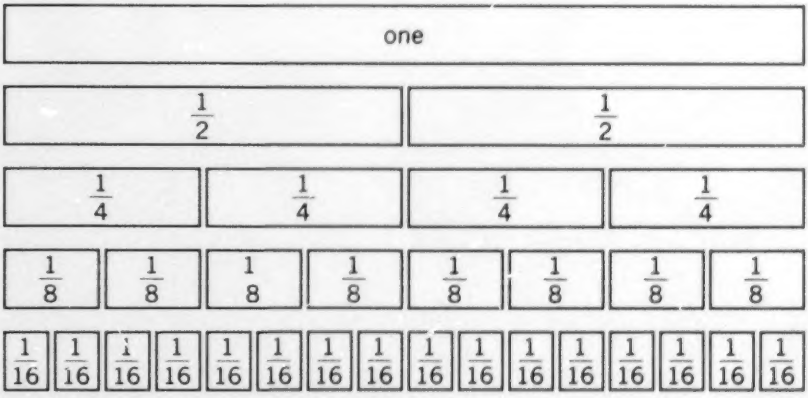
If purple is 1, then white is ____ and blue is ____ and ____ more.

- Fraction bar kits: These kits are student-made materials that reinforce the direct connection between concrete materials and fraction symbols. They can be used with fractions from the beginning level of developing understanding of the symbols through work with equivalent fractions to operations with fractions.

The first activity consists of making the fraction bar kits. Each student needs five strips of construction paper, each strip cut from a different color. (Strips of 3" x 18" can be cut easily with a paper cutter.) Explain to the students that these strips are going to be used to make materials for studying about fractions. Ask, "What do we need to know first to be able to talk about fractions?" If the students' pre-fraction concepts are strongly developed, they will recognize the need for identifying the unit first. So the first step in the activity is to label one of the strips as the unit. Students should all use the same color and label the strip with one or more symbols for a unit: whole, 1, one.

With the unit defined, students can fold the other four strips to make halves, fourths, eighths, and sixteenths. Explicit directions can be given for making these fraction bars or students may be given a chance to explore different ways of folding. In either case, it is important that some discussion take place as to why the folding procedure produces the correct fraction.

"Folding the strip in halves and then in halves again makes four equal pieces. So each piece is equal to $\frac{1}{4}$." Students should label each with its numerical symbol on one side and its word symbol on the other side, except for the sixteenth pieces that do not have room for "one-sixteenth."



Each student now has a personal fraction kit. Pieces can be replaced with minimum efforts, and the kit can be expanded to include thirds, fifths, sixths, etc. with some experimental folding. Students should make their own kits; the connection between the symbols and the materials comes mainly from the construction of the kit itself. For example, cutting and labeling the pieces and seeing them compared to the others prevents common confusion about which fraction is greater, $\frac{1}{5}$ or $\frac{1}{6}$. Making the fraction kit builds the understanding that the more pieces in which the whole is cut, the larger the denominator of the fraction that describes the pieces and the smaller the pieces. The misconception that $\frac{1}{16}$ is greater than $\frac{1}{4}$ because 16 is greater than 4 comes from focusing upon the abstract symbols, not upon the concrete examples of those symbols.

After making the kits, the class can be divided into groups of four students each. Ask each group to write down four sentences that describe something they noticed while making the fraction kits. Each group's sentences then will be shared with the rest of the class. Discovery statements could range from, "It takes two $\frac{1}{2}$ pieces to cover the whole piece" to "A $\frac{1}{2}$ piece can be made from one and one-half $\frac{1}{3}$ pieces." If students have trouble forming sentences about their observations, have the whole class participate in writing one or two sentences to use as starting points.

Cover is a game played with fraction kits that helps students become more familiar with the fraction concepts involved. It can be played by two to six students with a fraction kit for each student and one cube with the six sides labeled as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{16}$ to use as a die.

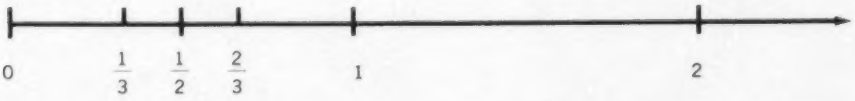
Cover

Each student starts with the whole strip. The goal is to be the first to completely cover this unit strip with other strips from the fraction kit. No overlapping pieces are allowed. Students take turns rolling the cube. The number face up on the die determines what size piece students may put on their whole strips. Students continue until one person has completely covered the strip. When the game nears the end and a person needs only a small piece such as $\frac{1}{8}$ or $\frac{1}{16}$ to cover up and win, rolling $\frac{1}{2}$ or $\frac{1}{4}$ won't do. The student must roll exactly what is needed.

Discussion time should be held after the game so students can share observations and ideas about game strategies. For example, why is there only one $\frac{1}{2}$ on the die.?

Students who need more of a challenge can use what they know about fractions of geometric regions to make a fraction kit based on the unit being a circle, a square, a triangle, a hexagon, etc. They should look for patterns as to which shapes can be folded into which fractions.

- Number lines: The number line is a more abstract representation of fractions. The number line is much closer to numerical symbols than it is to other concrete materials. Therefore, number lines are most useful when students have used other concrete materials to develop meaning for fractions and are beginning to work with symbols standing alone. Number lines are most useful in showing the relative values of fractions and whole numbers by showing their relative positions on a scale.



Students can make observations such as "$\frac{2}{3}$ is greater than $\frac{1}{2}$ because it is to the right of $\frac{1}{2}$ on the number line." Then they can verify this observation using manipulatives such as paper shapes or fraction bars.

Number lines are also useful when working with fractions greater than one and with equivalent fractions.

Models For Fractions Greater Than One

As students become familiar with the various manipulatives and the connections between the physical representations of fractions less than one, you can begin to introduce physical representations and language for fractions greater than one. The language should include both improper fractions and mixed numbers used interchangeably with the concrete representation. If the activity involves geometric regions (paper cutouts or drawings), having the whole regions divided into parts shows a direct connection between the improper fraction and the mixed number.

Each section is $\frac{1}{4}$ of the unit circle. So the picture is worth $\frac{11}{4}$, or 2 circles and $\frac{3}{4}$ more, or 2 and $\frac{3}{4}$ circles.

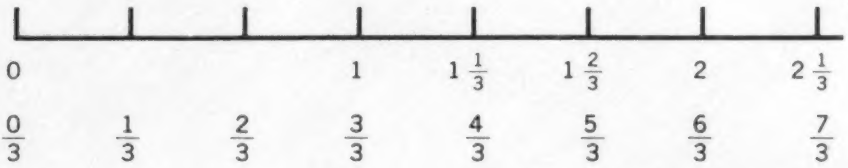
The symbol $\frac{11}{4}$ follows directly from the earlier use of $\frac{2}{4}$ and $\frac{3}{4}$ and the language used to count the amount in the picture: "one fourth, two fourths . . ., four fourths . . ., eleven fourths." The mixed numeral $2\frac{3}{4}$ combines two previous concepts: using whole numbers to describe pictures and using fractions to describe parts of whole. It matches the language that describes the pictures: "two circles and three-fourths more, or two and three-fourths."

Pattern blocks and tangrams can be used in the same way. Have each student make a pattern using different shapes. The pattern can be drawn on pattern block paper (Appendix C). Ask the students the following questions about their patterns:

If a hexagon (yellow) = 1, my pattern is worth _____.
If a trapezoid (red) = 1, my pattern is worth _____.
If a diamond (blue) = 1, my pattern is worth _____.
If a triangle (green) = 1, my pattern is worth _____.

If students are to the point of working with symbols, each can make an answer sheet for his or her pattern. The drawn patterns and answer sheets can be used as a class set of student-made task cards.

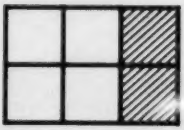
Number lines provide another visual link between the different symbols for fractions greater than one. Students can label points on the number line with both symbols.



Here again, the number line should be used after work with or in connection with more concrete manipulatives.

Models For Equivalent Fractions

Most students, after folding geometric shapes and making fraction kits, will have begun to see some equivalent fraction relationships such as two fourths making one half, even though they may not be able to record this idea with symbols. One of the advantages of using folding models is that equivalent fractions can be generated easily. If a student folds a rectangle in thirds, colors one third, and then folds the thirds in halves, the relationship of thirds and sixths will be shown directly when the paper is opened up:



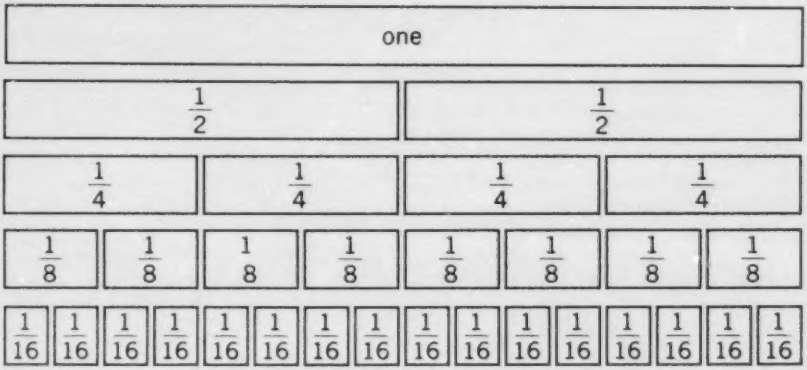
Have the students talk about the results and then translate their language into mathematical symbols, $\frac{1}{3} = \frac{2}{6}$, $\frac{4}{6} = \frac{2}{3}$.

Equivalent fractions should also be represented with collections of objects. This task is most easily done by bundling objects in the collection.

> Mrs. Parker has 6 pencils—2 were sharpened and 4 were not. What part of the collection of pencils is sharpened? ($\frac{2}{6}$) What part is not sharpened? ($\frac{4}{6}$) If Mrs. Parker groups the pencils by two's, how else can you describe what part of the pencils is sharpened? (One out of three bundles, or $\frac{1}{3}$.) What part is not sharpened? (Two out of three bundles, or $\frac{2}{3}$.)

Students can also use colored cubes to translate a geometric solid into a set of objects. In this way, students see that collections of objects represent the same equivalencies as parts of connected wholes.

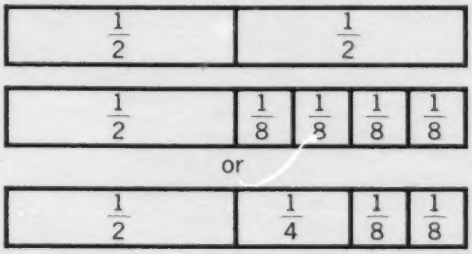
Fraction kits also offer direct visual representations of equivalent fractions.



Uncover is a game that develops concepts of equivalent fractions. It can be played by two to six students with a fraction kit for each student and one cube with the six sides labeled as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{16}$ to use as a die.

Uncover

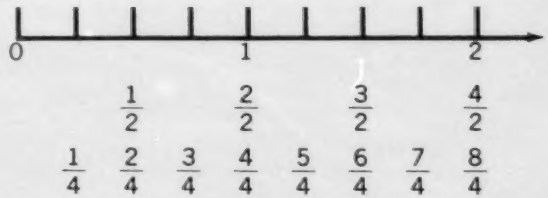
Each student starts with a whole strip covered with the two $\frac{1}{2}$ pieces. The idea is to be the first to uncover the strip completely. Students take turns. The fraction a student rolls on the cube tells what can be removed. Students may have to trade first so that they can do this. (That's the idea of the game, from a teaching point of view.) For example, if a student rolls a $\frac{1}{8}$ on the first roll, he or she has to replace one of the $\frac{1}{2}$ pieces with a number of smaller pieces whose total is equivalent to the $\frac{1}{2}$ piece. Extra pieces are provided in a box. (When this has been done to the satisfacion of the other players, the student then removes a $\frac{1}{8}$ piece.



Sometimes students may have to combine pieces to remove them, that is, two $\frac{1}{16}$ pieces if $\frac{1}{8}$ is rolled. The game continues until one player clears his or her strip.

Time should be allowed after the game sessions for students to share observations with the entire class. For example, do two pieces left on the board mean you have a maximum of two more rolls to win? What if the two pieces are $\frac{1}{8}$ and $\frac{1}{16}$? What if they are $\frac{1}{8}$ and $\frac{1}{8}$?

When students are familiar with representing equivalent fractions with a fraction kit or other manipulative materials, the symbols can be placed a few at a time on the number line to give another visual image of the equivalencies.



The number line can be made of cardboard and laminated so the symbols can be written and erased as needed. Other constructions for easily adding and removing symbols include hooks on the number line with numeral cards to hang on the hooks or adhesive-backed numeral cards to attach to a number line made of an adhesive strip.

In their work with the geometric regions, fraction kits, and number lines, students should be led to organize their observations so the pattern for equivalent fractions becomes apparent.

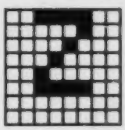
$$\frac{1}{2} = \frac{2}{4} \qquad \frac{1}{4} = \frac{2}{8}$$

$$\frac{1}{2} = \frac{3}{6} \qquad \frac{1}{3} = \frac{2}{6}$$

The rule for equivalent fractions—multiply or divide the numerator and denominator of the fraction by the same number—should come from the patterns observed when using concrete materials.

The following graph paper activities combine symbolic representation of equivalent fractions with pictorial representations:

- Have students use graph paper to show various fractional amounts in different forms. For example, in the following illustration, students were asked to make a "Z" by shading three-sixteenths of the rectangle.



- Have students start with 100 squares to make a design in which $\frac{1}{2}$ of the squares are red, $\frac{3}{10}$ are blue, $\frac{1}{10}$ are yellow, and $\frac{1}{10}$ are green.

Teaching To Improve Problem-Solving Skills

Recent research in problem solving indicates a few broad generalizations hold true. One generalization that can be directly controlled by the classroom teacher is: problem-solving skills are improved by incorporating them throughout the curriculum. The activities for developing fraction concepts presented in this publication are built around the problem-solving characteristics of understanding terms and concepts, noting likenesses and differences, visualizing, making generalizations, or using a variety of methods.

Teaching about fractions, or any other topic in elementary mathematics, through the use of manipulatives and exploratory activities can strengthen students' problem-solving abilities as well as increase their knowledge of specific content. In early childhood education, problem solving can be woven into the curriculum as learning techniques and applied whenever the situation demands, not as separate topics of pattern finding, estimation, or problem solving.

Instruction to improve problem solving cannot be isolated from the usual topics—place value, addition, multiplication, fractions, decimals. These topics must be developed through activities that require students to use the problem-solving skills we want them to develop. Students must be given many opportunities to analyze situations, to make generalizations from observation of patterns, to structure similar situations, and to look for the patterns that verify their generalizations. They must be allowed sufficient time to discuss their conclusions and reflect upon the way those conclusions were made. A recent doctoral study at Ohio State University found that a few problems plus discussion between students and teacher was more effective than students' working many problems on their own.

Discussion time also provides valuable information for the teacher as to how well the student understands the topic and what misconceptions might have been formed. When talking with students, react to their reponses as useful information rather than as merely right or wrong answers.

Use the following interview strategies to encourage a student to discuss a concept with you, thereby providing information about the level of understanding:

- Ask students to use manipulatives, draw a picture, or tell a story to represent an idea in concrete form.
- Change the line of questioning to a related concept or to a different level of difficulty.
- Use specific examples or, especially, counter examples to allow the student to recognize a misconception.
- Represent a rule with a pattern, then guide the student with leading questions to generalize the rule from the pattern.

These techniques can be used not only with fraction activities but throughout the entire elementary mathematics curriculum. With this type of instruction, the teacher generates an active and therefore more interesting role for the learner.

Appendixes

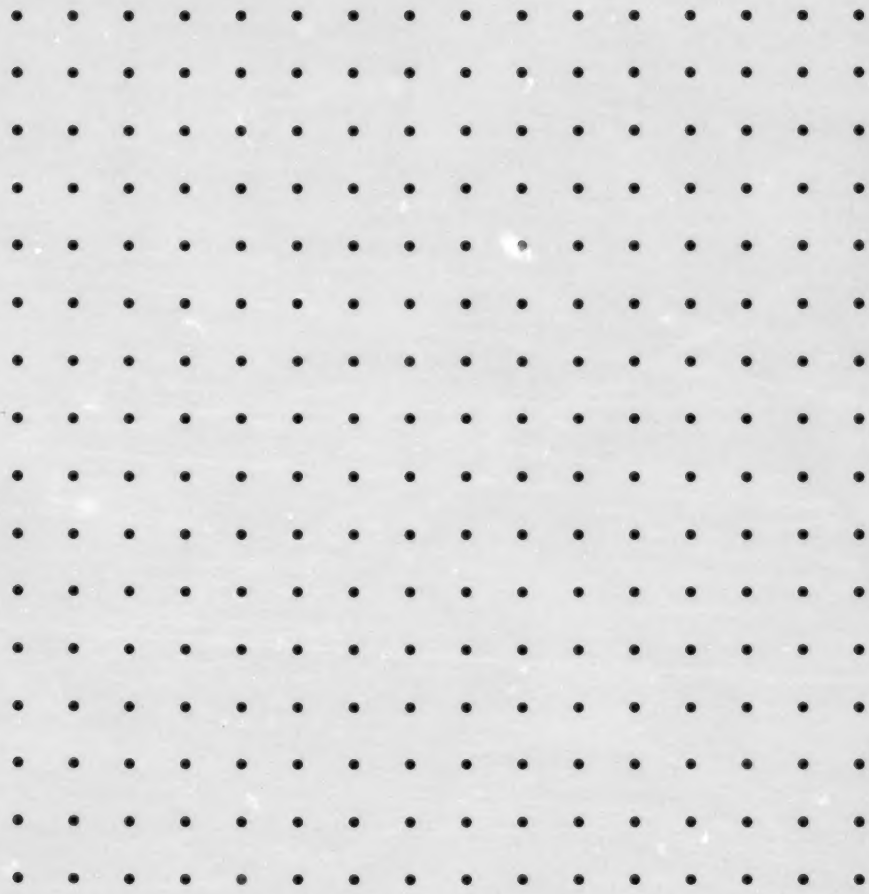
A. Dot Paper

B. Centimeter Graph Paper

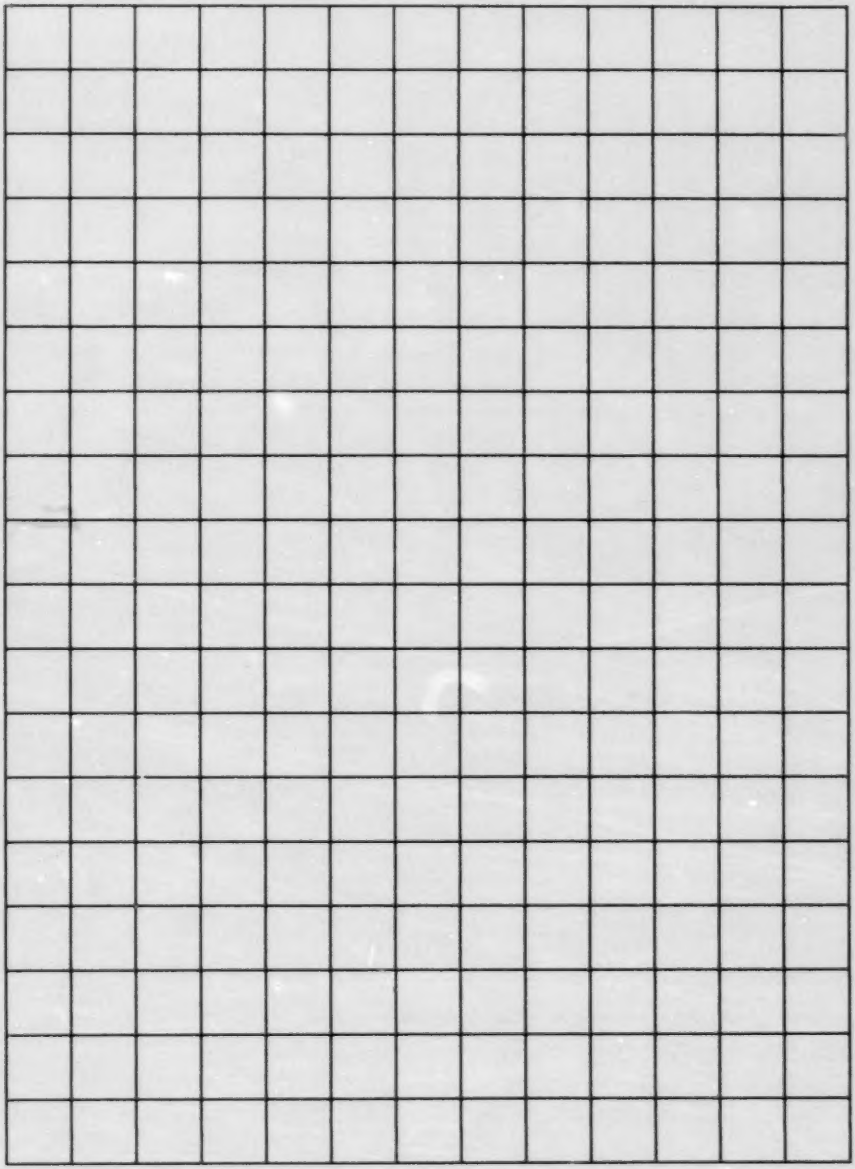
C. Pattern Blocks

D. Tangram

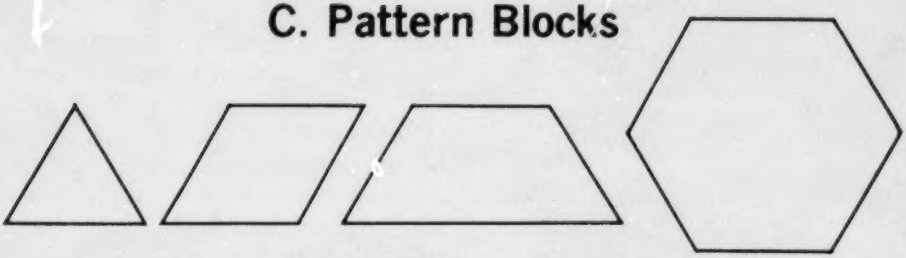
A. Dot Paper



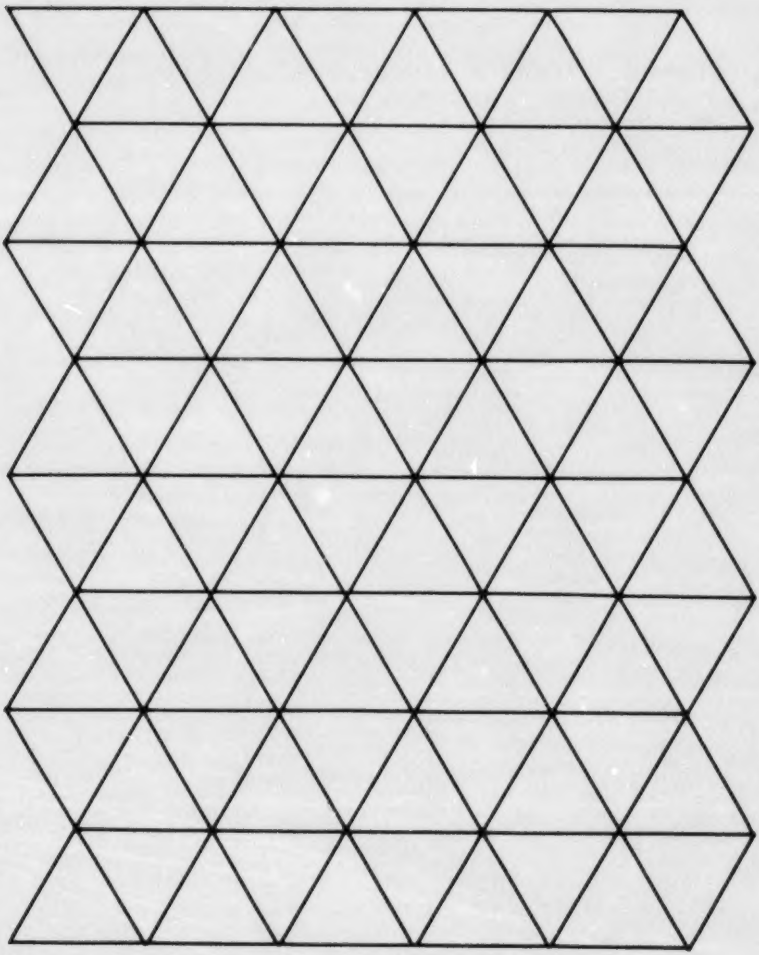
B. Centimeter Graph Paper



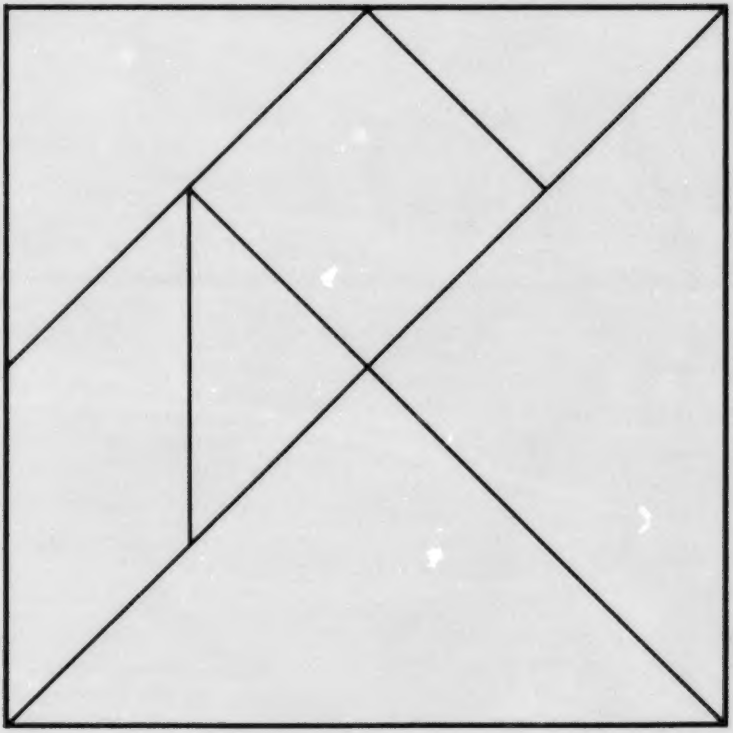
C. Pattern Blocks



Pattern Block Paper



D. Tangram



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